Ellipse-fitting approaches based on medial representation

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Abstract[1]

This paper presents two approaches related to the area of computer vision, where shape modelling (representation) is an essential part of every system. Specifically, the problem of fitting ellipse fitting is approached. The developed methods deal with elongated round-ended shapes, and represent them with ellipses based on medial representation. The evaluation shows that the proposed approaches outperform the state-of-the-art techniques on the examined dataset.

1. Introduction

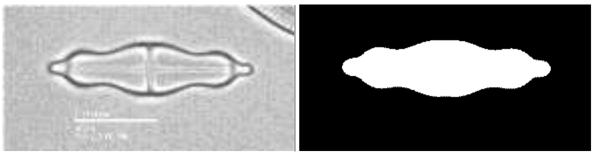
Representing the given shape with simple primitives that preserve its important structural characteristics is a highly researched topic that has applications in pattern recognition, surveillance and monitoring, computer graphics, mechanical engineering etc. [5]. Particularly, ellipses contain information about the elongation of the shape and orientation of its major axis [3].

The remaining of the paper is structured in the following way. Section 2 makes a brief introduction to the domain of shape representation, and is followed by Section 3, where the existing approaches to ellipse fitting are described. Section 4 provides a summary of the proposed techniques [2] and [3]. Section 5 discusses the experimental results by comparing state-of-the-art approaches [7] and [11] with the proposed [2] and [3].

2. Recall on shape representation

Shape plays a special role in computer vision [1]. By definition, 2D shape is a binary representation of an object, where *1s* correspond to its extent, and *0s* – to the background, or vice versa. As an example, see Figure 1 which shows an image of the diatom[2] (a), and its shape (b).

Shape modelling, or alternatively shape representation, is a fundamental concept in the domain of computational shape analysis. It is defined as a process of describing an object of interest w.r.t. preservation of its important characteristics.



(a) (b)

Fig. 1. Image of a diatom[2] (a), and its corresponding shape (b)

The approaches to shape representation can be classified to *contour-based*, since dealing with the outline of an object, and to *region-based*, since taking the outline as well as inner points into consideration [1]. The developed approaches [2], [3] belong to the second group of methods, and use as a basis the Distance Transform [16].

3. Overview of approaches for ellipse fitting

Hereafter, the problem of shape representation with ellipses will be considered from the perspective of fitting the ellipses to the given object. The existing literature distinguishes the following ways to tackle it:

(a) find the minimum ellipse that covers the whole object;

(b) find the maximum ellipse that is inside the object;

(c) minimize the total deviation between the object and an ellipse;

(d) find a set of ellipses covering the given object.

Depending on the category of the approach, discussed in Section 2, the object will be defined by the boundary of a shape (for contour-based), and/or as a shape (for region-based).

Following the categorization of Wong et al. [5], from conceptual perspective there exist three major branches of ellipse-fitting methodologies: (m1) Least-Squares - based, (m2) voting scheme-based, (m3) uncategorised, statistical or heuristic, combined techniques. The majority of them descend either from groups (1), or (2).

The idea of Least-Squares (LS) is to minimize some error function that measures the distance between points of the predicted ellipse and the points of the original data. The

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[2] taken from the ADIAC Diatom Dataset - http://rbg-web2.rbge.org.uk/ADIAC/pubdat/downloads/public_images.htm, Accessed: 2015-10-22

advantage of this approach lies in its linearity which enables real-time performance. The disadvantage is the sensitivity to noise and outliers that highly impact the result. Various metrics were proposed to enhance the robustness of the method. The substantive comparison of error of fit (EOF) functions was conducted by Rosin and can be found in [6]. Fitzgibbon et al. [7] made a crucial impact to LS. Direct least square (DLS) fitting considers elliptical constraint in normalization factor, and solves the problem in a non-iterative manner. It raised the new wave of research in this direction [8], [9], [10].

In contrast, voting scheme-based (clustering, grouping) approaches are stable to noise, though they are computationally more expensive. One of the well-studied methods in this area is the Hough transform (HT). Its ground principle is analogous to the locus. Points that correspond to the given shape contribute to the bin with the same shape parameters in an alternative space, and then the bins which are above the threshold are selected. The fact that HT is robust, but computationally expensive and time-consuming motivates researchers to further develop this approach, by using the geometrical properties of points [13], tangents of ellipses [11], symmetry [12], or by lowering the resolution of the data with image pyramids [14].

While considering the uncategorized techniques, it is worth mentioning hybrid approaches. They make an attempt to combine the advantages of multiple schemes: LS and HT [11], [13]; Watershed Transformation (WT) with LS [15].

4. Methods

In general, the developed approaches [2] and [3] employ the function of thickness change as a descriptor for splitting the given shape into elliptical parts. This fact fits them to the (m3) methodological group of methods.

The idea behind [2] is that implicit representation of an ellipse requires only three parameters to compute the positions of its points. Namely, these parameters are the lengths of semi-major and semi-minor axes, and position of the ellipse centre. For this purpose Gabdulkhakova et al. proposed to analyse the thickness profile, which is a 1D function that computes local thickness of the shape along its, medial representation w.r.t. some distance measure.

Medial representation, or alternatively skeleton, can be described as a set of points inside the region that are equidistant from its borders, and correspond to the centres of maximum circles fitted inside this region. The traditional approaches to obtain a skeleton include, but are not limited to Distance Transform (DT) [16], Medial Axis Transformation (MAT) [4], and thinning [17]. In case of [2], DT with City-Block distance metric was selected to approximate the local thickness of the shape since: (1) it provides not only the positions of the skeletal points, but also their distance to the borders; (2) City-Block is a D_4 metric, and produces the thickness values closer to reality than Euclidean, which is D_8.

The parameters of the ellipse are detected from the thickness profile with the help of the properties of its first and second derivatives. For a single ellipse, thickness profile has a distinct point with the highest value, local maximum, and two points with the low values, local minima. In this manner, if the centres of multiple ellipses do not coincide, then the local maxima of shape's profile indicate the positions of their centres, and the corresponding DT value equals the length of their semi-major axes. Local minima are assumed to be the points on the ellipse, and are employed for computing the length of their semi-minor axes w.r.t. implicit representation of an ellipse.

In addition, the transition between two ellipses within one shape does not always cause local extrema. Therefore, second derivative is intended to detect the point of the profile, where its slope was sufficiently changed.

The same approach for finding ellipse parameters was employed in [3], with the difference that the skeleton was obtained with thinning [17], and the thickness profile was represented by longitudinal and latitudinal values along this skeleton. The later representation has an advantage of being invariant to deformations.

5. Discussion of the results

In experimental part we compare the proposed approaches [2], [3] with the state-of-the-art methods [7][3]and [11][4]. The test data contains images from diatom dataset[2], as well as synthetically generated data.

Synthetic data is represented by challenging cases of the shapes, such as very elongated (d), with high negative curvature (a), high positive curvature (f), and ellipses which are approximated by polygons (e). The idea is to check how different methods cope with extreme situations.

Diatoms that compound the dataset are symmetric objects that either have elliptical shape, or can be represented by multiple overlapping ellipses. The motivation to use these data is to check the possibility of each method not only to find a best single ellipse fit, but also to approximate the given shape with several ellipses.

As an input data, methods [7] and [11] received the set of boundary points of the shape, whereas our method employed the whole region of the shape.

The results are shown in Fig. 2. First row (a) – (f) correspond to the given shape, where (b) and (c) are from diatom dataset, and the remaining are synthetically generated. Second (a1) – (f1), third (a2) – (f2), and forth (a3) – (f3), fifth (a4) – (f4) rows demonstrate the results of Fitzgibbon et al. [7], Cicconet et al. [11], Gabdulkhakova et al. [2], and Gabdulkhakova et al. [3] correspondingly. In figures (a1) – (f1) and (a2) – (f2) the resultant fits are highlighted with yellow. In figures (a3) – (f3) the original region is shown in white, green ellipses

[3] Implementation of the algorithm is adopted from http://research.microsoft.com/en-s/um/people/awf/ellipse/fitellipse.html

[4] Implementation of the algorithm is adopted from https://bitbucket.org/cicconet/triangles_matlab/src

are detected by the proposed algorithm [2], blue ellipses are fitted w.r.t. to the points of medial representation that lie between focal points of the green ellipses.

The general difference between compared approaches is that [7] and [11] focus on minimizing the total deviation between given shape and an ellipse fit, allowing the latter not to strictly be inside the contour. In contrast, the methodology described in [2] detects such parameters of ellipse that it will be enclosed in a given region.

Evaluating by the number of fitted ellipses shows that, on one hand, DLS does not consider the case of several overlapping ellipses. Thus, given a composite object it fits a single ellipse with parameters that minimize the distance measure (a1), (b1), (c1) and (f1). On the other hand, for synthetically generated ellipse (d1) and ellipse approximated with the polygons (e1) the algorithm shows very good results. In contrast, hybrid approach [11] is able to find multiple ellipses that can fit the shape. Method requires the specification of several parameters by user such as estimated maximum number of ellipses, range of semi-major and semi-minor axes lengths. In present experimental setup it does not produce stable results – the algorithm provides different fits for the same input data. Moreover, elongated ellipse is not detected (d2). As opposed to [11] the proposed method does not require a priori knowledge about the parameters and number of ellipses. In comparison to [7], it is possible to fit multiple ellipses inside the given contour. The performance can be further improved by substituting the DT with more robust thickness computation method. The results of [3] highly depend on the original skeleton, since computing the latitudinal values as normal to skeleton at the corresponding point. Therefore, it is more dependent on small perturbations (d4) and polygonal approximations (e4) than [2].

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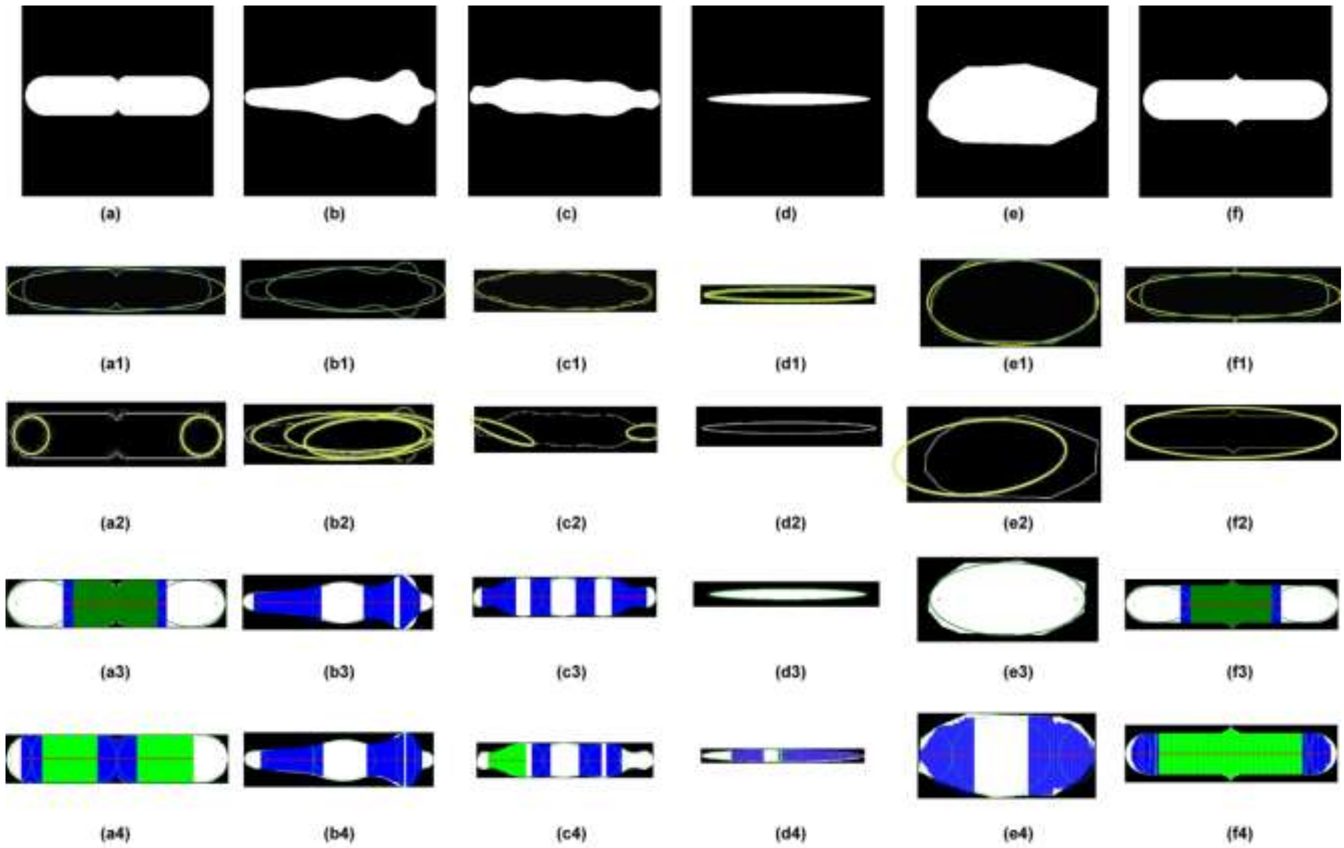

Fig. 2. Comparison of the results produced by [7], [11], [2], and [3]