Haezendonck - Goovaerts risk measure for optimal portfolio. Correlation α parameter on shape and position of effective portfolio boarder

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Abstract[1]

Haezendonck - Goovaerts risk measures overview was presented. Numerical experiment comparing results of calculating Haezendonck - Goovaerts risk measures was made. This experiment is true for portfolios built for specific family shares with risk aversion degrees α varying by constructing curves "risk-return" approach, in analogy with "portfolio theory" G.Markowitz

1. Introduction

The theoretical literature of risk management and decision-making under uncertainty sometimes can contains criticism of certain risk measures, for example, even such common as VaR. In this case, it is because the VaR works badly for random variables whose distribution is different from normal. For remove the potential problems of incorrect assessments in the future, it was defined axiomatically coherent risk measures. Coherent measures include also Haezendonck - Goovaerts risk measure. This risk measure has been introduced into consideration in the early 2000s.

The report provides a definition of Haezendonck - Goovaerts risk measure and performed numerical experiment compared the results of calculating Haezendonck - Goovaerts risk measure for a set of portfolios constructed for a particular family shares with variation risk aversion α using the plotting of "risk-return" similar approach "portfolio theory" of G.Markowitz.

2. Publishing

Normalized Young function – is function Φ: [0 +∞)□[0 +∞), which satisfies the following properties:

1) continuity;

2) non-decreasing;;

3) Φ(0)=0, Φ(1)=1, Φ(+∞)= +∞;

4) convex.

From a financial perspective, the function Φ is perceived as a loss function for insurance premiums. In mathematically calculated, it will be a function of the distortion. Risk measure, calculated on distorted probabilities, began to be used in the actuarial calculations and in determining insurance rates recently.

Let X denotes as non-negative bounded random variable representing the magnitude of random loss or gain.

The parameter α is related to one correspondence with the degree of loss aversion, with α = 0 investor should be indifferent to the loss, a value close to 1 - there is a case with a maximum loss.

The basis of determining the Haezendonck - Goovaerts risk measure is the value $\pi_\alpha(X,x)$, Defined as follows [1]: Let α ∈ [0; 1), x<sup(X) - a number, $\pi_\alpha(X,x)$ - This is the only root of the equation

$$E\left[\Phi\left(\frac{(X-x)_+}{\pi_\alpha(X,x)-x}\right)\right]=1-\alpha \qquad (1)$$

Where E - the expectation, $a_+ = (a+|a|)/2$.

Haezendonck-Goovaerts risk measure is the value

$$\pi_\alpha(X) = \inf_{-\infty<x<\sup[X]} \pi_\alpha(X,x) \qquad (2)$$

Properties of $\pi_\alpha(X)$ [3]:

1. Measure $\pi_\alpha(X)$ depends only on the distribution function *X*.
2. Measure $\pi_\alpha(X)$ is monotonic on *X* on the cone Θ,, i.e. if *Y,X* ∈Θ and *Y*≤*X*, then $\pi_\alpha(Y) \le \pi_\alpha(X)$. .

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3. Measure $\pi_\alpha(X)$ is positive homogeneous on a cone Θ, i.e. $\pi_\alpha(\lambda X) = \lambda\pi_\alpha(X)$ if $X \in \Theta$.
4. Measure $\pi_\alpha(X)$ is subadditive on the cone Θ, i.e. $\pi_\alpha(X+Y) \le \pi_\alpha(X) + \pi_\alpha(Y)$ if $Y, X \in \Theta$.
5. Measure $\pi_\alpha(X)$ is convex on the cone Θ; if the function Φ is strictly convex, then measure $\pi_\alpha(X)$ is also strictly convex on the cone Θ.
6. Measure $\pi_\alpha(X)$ is shift invariant on cone Θ, i.e. $\pi_\alpha(X+a) = \pi_\alpha(X) + a$.
7. $\pi_\alpha(c) = c$ if $X = c$ - deterministic value.
8. $\sup(X) \ge \pi_\alpha(X) \ge E[X]$ at any $\alpha \in [0:1)$.
9. Haezendonck-Goovaerts risk measure is compatible with the ratio of the convex order and with the stochastic dominance of the first order.
10. $P[X > \pi_\alpha(X)] \le 1 - \alpha.$
11. 13. If a sequence of nonnegative random variables X_n converges strongly to X, and $\sup X_n < A$ for some A, then $\overline{\lim}\, \pi_\alpha(X_n) \le \pi_\alpha(X)$.

Risk measure is called a coherent risk measure, if done properties 2,3,4,6. Thus, Haezendonck-Goovaerts risk measure is coherent risk measure.

The algorithm for computing measures Haezendonck-Goovaerts risk measure in the discrete case is as follows [3]:

Let the variable X has a discrete distribution: the possible values $0 \le a_1 < a_2 < \ldots < a_n$ are associated with probabilities $p_1, p_2, \ldots, p_n$.

1) For each of the intervals $[a_1, a_2], \ldots, [a_{n-1}, a_n]$ value is calculated $\pi_\alpha(X, x)$.

2) From these local minima, we select the smallest minimum.

Markowitz's efficient portfolio - a portfolio with a maximum permissible expected return for a given level of risk. A set of efficient portfolios is called the effective set of portfolios. The optimal portfolio is called a portfolio that best meets the preferences of the investor by the ratio of return and risk. [4]

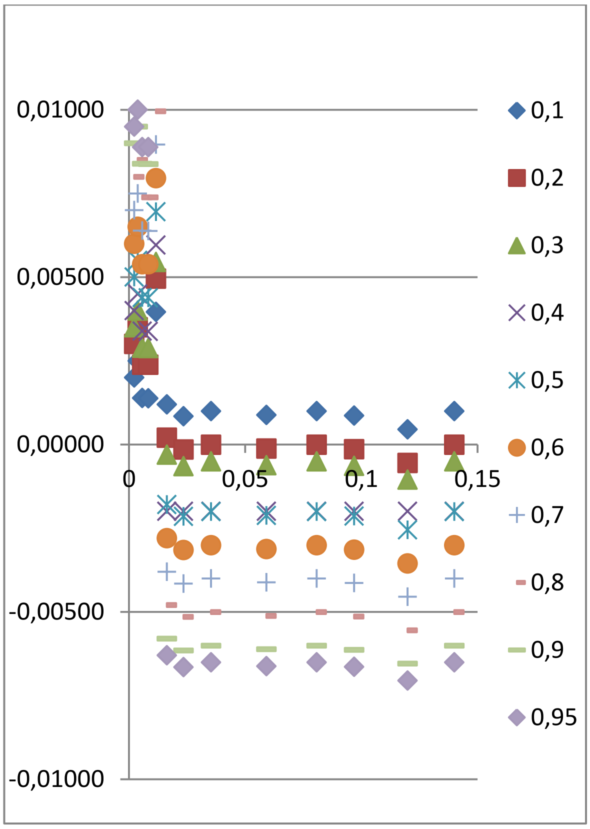

Fig. 1.

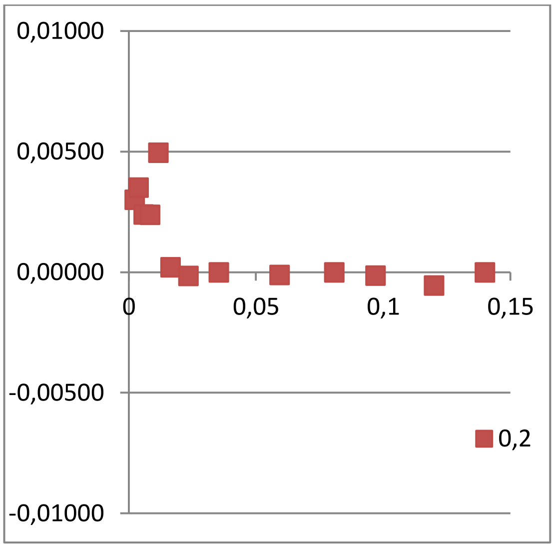

Fig. 2.

For a set of portfolios constructed for a particular family shares the points obtained in the plane "measure of the average yield - Haezendonck - Goovaerts risk measure" by varying the degree of risk aversion can be illustrated by the graphs of dependencies between yield curves and a Haezendonck - Goovaerts risk measure (Figure 1). The the graphs correspond to the values of the X-axis value of the Haezendonck - Goovaerts risk measure, and the value of y - corresponding yield portfolios at values calculated risk measures.

In the resulting chart, the maximum amplitude values of the measures of risk is achieved by the curves constructed using the highest values of the parameter α. It may be noted that the construction of the curve as it turns "fit" one inside the other.

General view of the efficient frontier obtained at one of the values of α is shown in Figure 2.

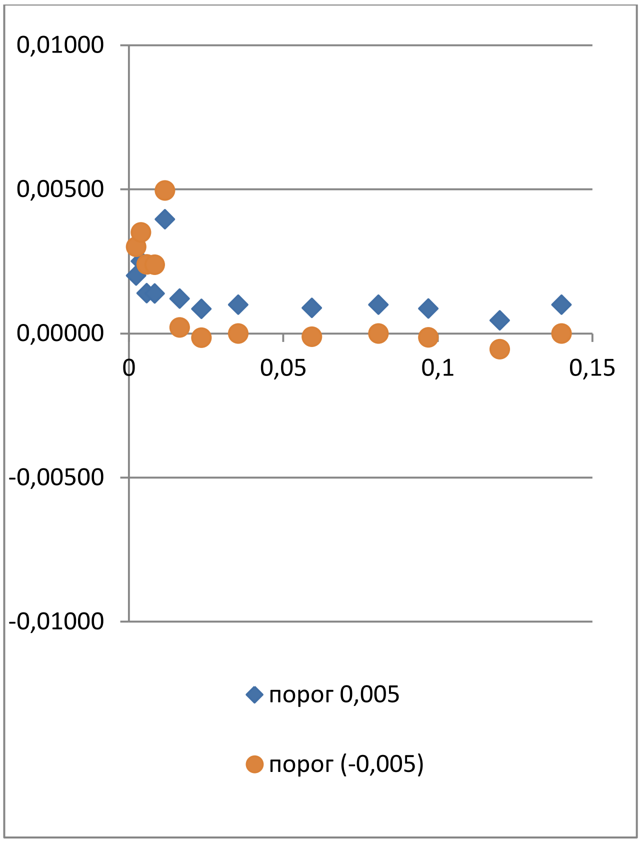

Fig. 3.

The graph (Figure 3,) effective border built as x (thresholds losses (-0.005) and (0.005). It can be seen that achieved the highest and lowest values of x yield change has no significant effect, however, there has been a shift and the changing nature of the curves (upward and downward).

Calculations of Haezendonck - Goovaerts risk measure and building an effective border were carried out in the software package MATLAB.

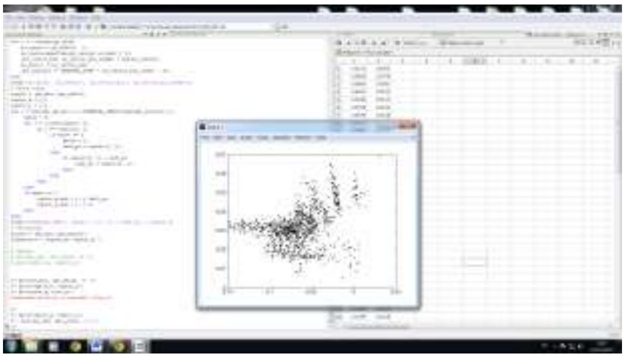

Fig. 4.

Over the time, the optimal portfolio choice questions and preferences of various risk measures involve research and economic sphere. This article provides an overview of Haezendonck - Goovaerts risk measure, the approach to the calculation of this measure for discrete risks. Numerical results compared to the results of calculation of Haezendonck - Goovaerts risk measure portfolios for a set at varying degrees of risk aversion α by constructing curves "risk-return" by analogy with "portfolio theory" G.Markowitz.

3. Conclusion

The main results:

- Haezendonck - Goovaerts risk measures overview was presented
- Haezendonck-Goovaerts risk measure is coherent risk measure.
- The maximum amplitude values of the measures of risk is achieved by the curves constructed using the highest values of the parameter α.
- There is a shift and the changing nature of the curves (upward and downward) due to changing the value x (thresholds losses).

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