The features of cutting plans design for different cutting technologies

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Abstract[1]

In this paper we consider different technologies used for cutting plan design. The features of cutting plan design for resource-saving technologies are considered. On the example of one of extreme cases for a rectangular cutting plan we see that ICP-technology is being the resource-saving by such criteria as non-productive waste of material, the length of a cut, and the number of in-cuts towards widely used by modern CAD/CAM systems GTSP-technology.

1. Introduction

Using of resource-saving technologies is actual nowadays. The amount of industrial waste depends on:

- manufacturing tolerances on the lip;
- cuts and bridges between the details;
- a combination of configurations of mutually adjacent details;
- multiplicity of workpiece material and size dimensions (especially noticeable when cutting large details).

There are the following ways to reduce the waste of material during cutting process:

- recycling;
- tightening of allowances;
- cuts combination;
- reduction of idle passes time.

B It was 1949 when the first publications on linear programming appeared. In 1951 the first issue of [1] appeared. This monograph dealt with problems of using the methods of linear programming for optimal guillotine cutting. The development of enterprise automation lead to appearance of technological equipment with numerical programming control (NPC) used for cutting off the details from a sheet material such as the machines of gas (oxygen), plasma, laser, and EDM cutting. The new technologies allow cutting the details by any trajectory with sufficient accuracy.

If we remove the requirement to cut off only by looping straight cuts then we may significantly reduce the waste of material. This fact became the impulse for a great number of papers, for example, [2–5] on non-guillotine cutting and its optimization by different enterprises and on the different stages of automation.

2. Classification of Routing Problems for a Cutter

Unlike the guillotine cutting plan the non-guillotine one does not define the program for cutting. The constructing of cutting automaton control program for a given cutting plan is a separate problem. In paper [6] the following classification of routing problems for a sheet cutting is suggested.

Generalized Travelling Salesman Problem (GTSP): the cutter successively moves by the contour of each detail. The possible incut points are fixed. This technology does not permit the combination of cuts (fig. 1).

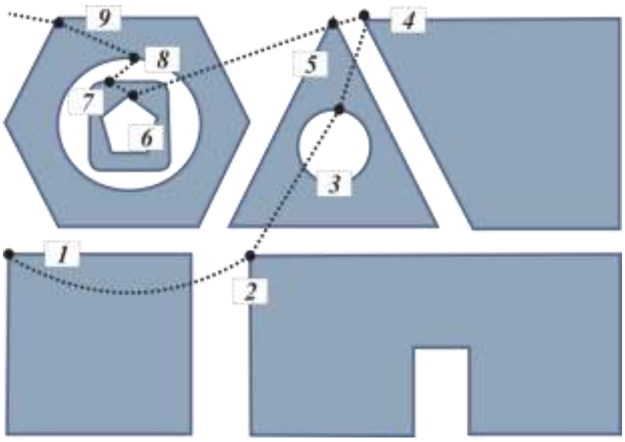

Figure 1. The example of cutting plan for using of GTSP-technology

Continuous Cutting Problem (CCP). The details are cut off consequently. The incut point may be at any part of a contour, and pass to another contour is realized only after the current one is cut off. As well as GTSP this technology does not allow combination of fragments of cutting contours. The optimal path be also the solution of generalized TSP on the set of incut points with

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precedence relations due to nesting of one contours inside the others (fig. 2).

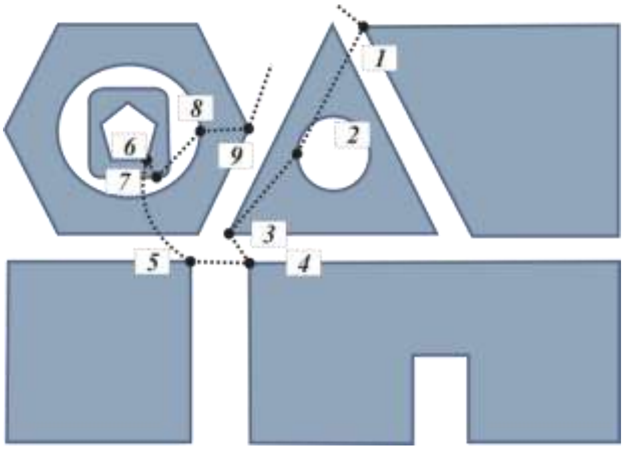

Figure 2. The example of cutting plan for using of CCP-technology

Endpoint Cutting Problem (ECP). The instrument incuts and goes to another cutting plan fragment in the given points of a contour frontier. The combination of contours is allowed. This leads to partial cutting of some contours. The sequence of passing of components is defined by solution of a generalized TSP on the digraph of possible transitions between these components with precedence constraints taking into account the nesting of components into contours of other components. As far as due to combination of fragments of contours the number of components is less than the number of details, then the size of ECP salesman problem in common is less than size of GTSP (figure 3).

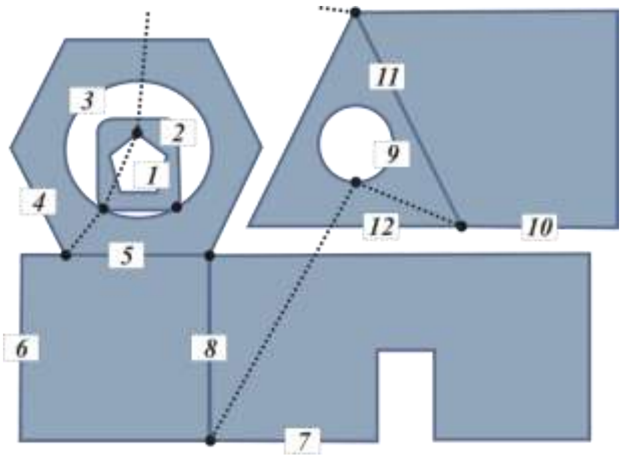

Figure 3. The example of cutting plan for using of ECP-technology

Intermittent Cutting Problem (ICP). The common case of cutting problem when combining of contours is allowed, and there are no constraints on the choice of incut points (figure 4).

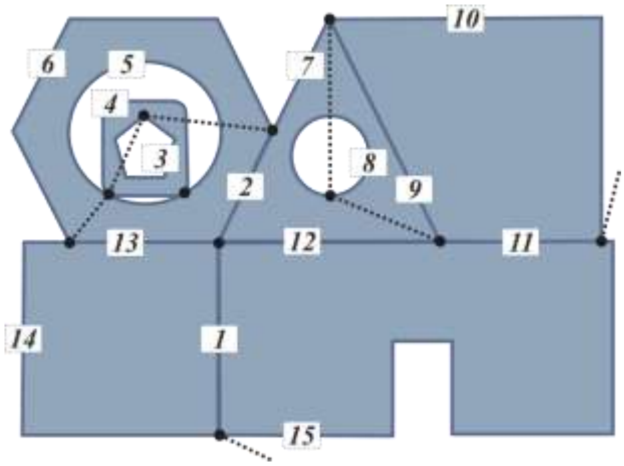

Figure 4. The example of cutting plan for using of ICP-technology

To realize this technology one needs the representation of cutting plan as a plane graph. Unlike ECP components are not to be Eulerian. The construction on a path to cut off each component is reduced to construction on ordered Eulerian covering for the given component by OE-trails [7, 8]. The order of routing of the components is defined by a solution of generalized TSP on the digraph of allowed transitions between components with precedence constraints according to the nesting of one components to contours of another ones. The deristriction of component to be Eulerian allow rise their number by overlapping fragments of contour parts and, consequently, decrease the number of components. Hence, the size of ICP salesman problem is not greater than size of ECP salesman problem [9].

GTSP and CCP technologies differ by sizes and values of elements of matrices which are the distances between the contours. These technologies assume only the optimization of idle passes and cutting programs are broadcasted up to the incut point. Here the length of a cut is equal to the sum of lengths of cut contours. Also the gap between details should be provided. This leads to additional waste of material. CCP technology in comparison with GTSP technology allows to shorten the time for idle passes between the incut points. There are lots of known heuristic algorithms for these technologies, for example, [10, 11]. The ways of rising the effectiveness of the exact algorithm for these technologies is considered in [12].

Technologies ECP and ICP due to combination of cuts allow reducing the waste of material, length of a hot-cut, the number and length of idle passes (see, for example, figures 1-4). However this essentially complicates the process of building the cutting program:

- the successive pieces of contours parts are not always the consistent elements of the cutting tool trajectory,
- the problem of definition of such a sequence of cutting claiming part cut off a sheet not to have additional cuts becomes not so trivial.

An attempt to estimate the effect of combining of cuts is made in [13]. In the same paper the absence of algorithms for routing problems for ECP and IC technologies is mentioned. Particularly, it is mentioned that an approach made in [14] claiming the solution of rural postman problem is hard to realize.

3. Stages of Cutting Process

Using of ECP and ICP technologies during the technological support of cutting processes for plane details assumes the following four stages:

1. cutting plan design;
2. abstracting of cutting plan to a plane graph;
3. solving of routing problem;
4. control program design.

The stages of constructing of cutting plan and interpretation of the defined route in terms of a cutter commands are the common for all technologies and good studied. The second and the third ones have some features. Let's consider them in details.

1. Cutting plan design means the definition of such a variant of cut details placement on a rectangular sheet or strip that allow to minimize waste and maximize the length of combined contours of cut-off details. The solution of such problems is suggested, for example, in [2-4].

2. Abstracting of cutting plan to a plane graph. To define the sequence of details cutting one does not need any information on the detail shape that is why all the curves without self-intersections and contiguities on a plane representing the shape of details are interpreted as graph edges, and all points of intersections and contiguities are to be graph vertices. To analyze the technological restrictions satisfaction one needs introduce the additional functions on set of vertices, faces, and edges of a graph obtained [9].

Let the model of cutting sheet be a plane *S*, the model of cutting plan be a plane graph *G*, its outer face be f_0. Let the set of vertices of graph G components that are not homeomorphic to a circle be the points of contact for three or more details of cutting plan, the corresponding fragments be the edges of graph incident to these vertices. Component homeomorphic to a circle be a loop. The plane graphs being the homeomorphic images of cutting plans for ECP and ICP technologies (figure 3-4) are presented on figures 5 and 6.

Let $\mathrm{Int}(J)$ be the set-theoretic union of inner faces for any subset $J \subseteq G$ (the union of all components S, J without outer face). If we consider that a cutter has moved by all edges of graph *J* then Int(*J*) be a part cut off a sheet.

Thus, using the known coordinates of vertex images for graph *G*=(*V*,*F*,*E*) and placement of details on cutting plan being the prototypes of graph edges one may easily construct ant route in graph *G*=(*V*,*E*) and interpret it as a path of a cutter.

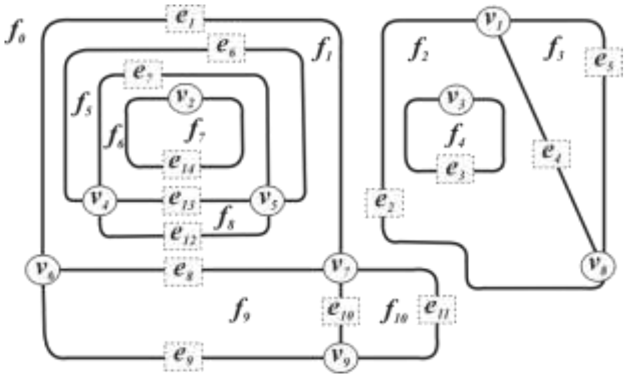

Figure 5. The example of abstracting of cutting plan to a graph for ECP-technology

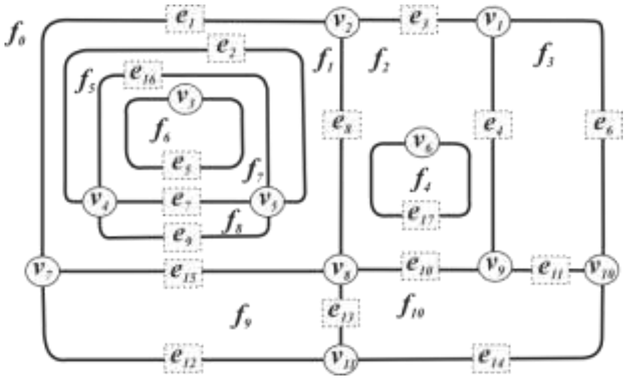

Figure 6. The example of abstracting of cutting plan to a graph for ICP-technology

It is shown in [7,8] that there is a route avoiding the passes of edges enclosed into the cycle of passed edges for a plane Eulerian graph. As for the common case when graph is not to be Euleran and disconnected one needs constructing some additional edges to make graph Eulerian. The minimal number of such edges (being the idle passes of a cutter) is equal to a half of odd degree vertices [15]. Such an addition of idle edges means the constructing of the shortest matching *M* on set of odd degree vertices. The length of an edge be the shortest way between two points on a plane (corresponding to incut points of a cutting plan) [15].

Frequently the cutting plan contains details with wholes, sometimes even some details are situated in these wholes etc. Hence, plane graph being the homeomorphic image of such a cutting plan is being disconnected.

Definition 7. *Let face* $f \in F(G)$ *be* ***separating*** *if graph* G' , $\{f\}$ *be disconnected.*

Let graph $\tilde{G}$ is being received of graph *G* by adding bridges between components at separating faces. Obviously, the obtained graph $\tilde{G}$ is plane connected graph and it is possible to get route with some restrictions for it. This route $M(G)$ can be obtained of route $M(\tilde{G})$ if all vertices incident to injected bridges are to be the ends of trails and beginnings of the next ones (i.e. these bridges are to be the idle passes).

Let's consider the way of constructing the bridges connecting graph *G* and having minimal summary length [9].

Algorithm *Bridging*

Input: plane disconnected graph *G*.

Output: plane connected graph $\tilde{G}$ and set *B* of added bridges.

Step 0. $\tilde{G} := G$; $B = \varnothing$.

Step 1. Define set C_F of separating faces.

Step 2. Run steps 3–6 for each separating face $f \in C_F$, and **stop** after that.

Step 3. Find set $S(f)$ of all components of graph *G* incident to face *f*.

Step 4. Construct the full abstract graph T. Its vertices are the components $S(f)$, and lengths of its edges are equal to distances between the corresponding components.

Step 5. Define the minimal spanning tree $T(\mathrm{T})$ of T.

Step 6. Add edges of MST to graph $\tilde{G}$: $E(\tilde{G} := E(\tilde{G}) \cup E(T(\mathrm{T}))$, $B := B \cup E(T(\mathrm{T}))$.

End of algorithm *Bridging*.

As for graphs corresponding ICP and ECP technologies these edges are shown by dot-lines on figures 3 and 4.

3. Solving of routing problem to construct the optimal paths with restrictions on the edges order. These restrictions directly imply technological restrictions on the order of details cutting such as part cut off a sheet does not require additional cuts [16], there are no intersections of cuts [17], the length of idle passes should be minimized [16], the number of incut points should be minimal [18] etc. To get a route for which part cut off a sheet does not require additional cuts one may use any algorithm for constructing of so called OE-covers to graph obtained on the stage of abstracting.

Definition. *Let the ordered sequence of edge-disjoint OE-trails*

$$C^0 = v^0 e_1^0 v_1^0 e_2^0 ... e_{k_0}^0 v_{k_0}^0, \quad C^1 = v^1 e_1^1 v_1^1 e_2^1 ... e_{k_1}^1 v_{k_1}^1, \ldots, C^{n-1} = v^{n-1} e_1^{n-1} v_1^{n-1} e_2^{n-1} ... e_{k_{n-1}}^{n-1} v_{k_{n-1}}^{n-1}$$

covering graph G and such that

$$(\forall m : m < n), \quad \left(\bigcup_{l=0}^{m-1} \mathrm{Int}(C^l)\right) \cap \left(\bigcup_{l=m}^{n-1} C^l\right) = \varnothing$$

*be called **OE-cover**.*

Algorithm M-OE-Router

Input: connected plane graph *G*, specified by functions $v_k(e), f_k(e), l_k(e)$, $e \in E(G)$, $k = 1, 2$; vertex $v_0 \in V(G)$ incident to outer face; matching *M* on the set of odd degree vertices.

Output: almost ordered set *C* of OE-trails of graph *G* being the OE-cover of graph *G*.

Intermediate data: for each $v \in V(G)$ the queue $Q(v)$ of incident edges $e \in E(V)$ ordered by decreasing of ranks; vertices $u, v \in V(G)$; edge $e \in E(G)$; a symbol # of trail ending; for each vertex $v \in V(G)$ mark

$$\mathrm{Odd}(v) = \begin{cases} \textbf{true}, \text{ if } v \in V_{Odd}, \\ \textbf{false}, \text{ if } v \notin V_{Odd}; \end{cases}$$

Step 0. <*Initialization*> $v := v_0$; $C << v_0$; for all $v \in V(G)$ define the value of mark $\mathrm{Odd}(v)$.

Step 1. <*Marking*> Define the ranks of all edges, vertices and faces of graph *G*. For each vertex $v \in V(G)$ form a queue $Q(v)$.

Step 2. If $\mathrm{Odd}(v)$ go to **Step 3**, otherwise go to **Step 4**.

Step 3. If $Q(v) = \varnothing$ go to **Step 7**, otherwise go to **Step 5**.

Step 4. If $Q(v) = \varnothing$ then **Stop**, otherwise go to **Step 6**.

Step 5. If $\mathrm{rank}(v) \leq \mathrm{rank}(M(v))$ go to **Step 7**, otherwise go to **Step 6**.

Step 6. Run the following operators:

$e << Q(v)$; /* move the first element from queue $Q(v)$ to variable *e* */

$v := u : e = \{v, u\}$; $C << e << u$; /* the next edge and vertex of a trail */

go to **Step 2**.

Step 7. Run the following operators:

$u = M(v)$; /* the next vertex *u* is a mate of vertex *v* */

$\mathrm{Odd}(u) := \mathrm{Odd}(v) :=$ **false**; /* delete vertices u, v from list V_{Odd} */

$C << v << \# << u; v := u$; /* ending the current trail and beginning the new one */

go to **Step 2**.

End of algorithm *M-OE-Router*

The proof of this algorithm effectiveness for plane graphs without bridges is given in paper [15]. At the same paper it is shown that computing complexity of algorithm does not exceed the value $O(|E(G)| \cdot \log |V(G)|)$. Let's admit that connected plane graphs being the images of cutting plans do not contain any bridges. Hence, algorithm *M-OE-Router* constructs a route with minimal length of additional edges for them. If the initial graph was connected by algorithm *Bridging* it must have some bridges. In this case to run algorithm *M-OE-Router* one needs put the bridges twice.

4. Control program design for a cutting process using the path defined by algorithm of abstracted routing problem solution. Here the reverse replacement of abstract edges of plane graph by a system of commands to a cutting automaton holds. This replacement allows the movement by curves on a plane according to the detail shape.

4. The Features and Differences of Cutting Plan Design for GTSP and ICP Technologies

Let's consider in details some features for cutting plan design for different cutting technologies.

The design of cutting plan for GTSP-technology means the cutting one contour after another. That is why if *D* be the width of cutting then without cuts combination the details should be placed on the distance not less than 3*D* (figure 7). In opposite, after combining the cuts details should be placed on distance *D* (figure 8).

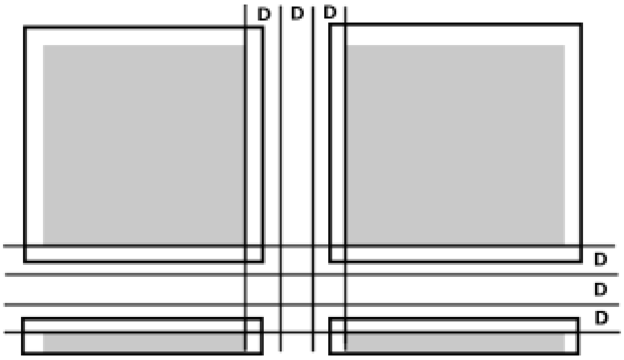

Figure 7. Placement of details for GTSP-technology. The minimal distance between details without combination of cuts

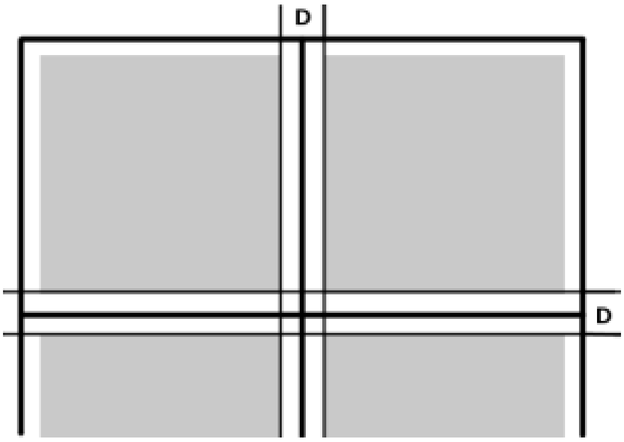

Figure 8. Placement of details for ICP-technology. The minimal distance between details with combination of cuts

Let's consider the extreme case when one needs to place $m{\cdot}n$ squares of size *s*. The optimal placement of such details is shown on figure 9.

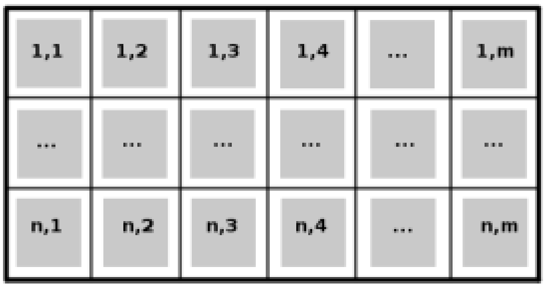

Figure 9. The cutting plan consisting of $m{\cdot}n$ square details

If there is no combination of cuts the **non-productive waste of material** be equal to

$$3D(n-1)+3D(m-1)=3D(n+m-2).$$

After combining the cuts the non-productive waste be

$$D(n-1)+D(m-1)=D(n+m-2).$$

Thus, non-productive waste can be shorten up to 3 times due to combining the cuts.

Let's define the **length of a cut** *L* without idle passes and width of a cut. Without combining the cuts it is greater than the product of the square perimeter and the number of squares

$$L \geq (4{\cdot}s){\cdot}(m{\cdot}n).$$

After combining the cuts we have

$$L=(m-1){\cdot}(n-1){\cdot}2{\cdot}s+2{\cdot}m+2{\cdot}n=2{\cdot}s{\cdot}n{\cdot}m+2{\cdot}s=$$
$$=2{\cdot}s{\cdot}(m{\cdot}n+1).$$

So the length of a cut after combining the cuts may be shorten for more than 2 times.

The number of incut points is equal to $|V_{odd}|=m{\cdot}n$ i.e. equal to the number of rectangles if there is no cuts combination. As for combined cuts all incut points (being the odd degree vertices of homeomorphic graph image of a cutting plan) are placed on outer bound of a cutting plan and their number is one less than the number of rectangles in the corresponding row (column), i.e.

$$|V_{odd}|=2{\cdot}(n-1)+2{\cdot}(m-1)\,.$$

Hence, using the cuts combination the number of incut points is an order of magnitude lower than in the case without combination of cuts.

5. Conclusion

Thus, ICP-technology is being the resource saving by such criteria as non-productive waste of material, length of a cut and the number of incuts in comparison with widely used by modern CAD/CAM systems GTSP-technology.

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